

Geometric and Economic Considerations in the Design of Flat Storage Systems

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ABSTRACT

The geometric shape of flat storage systems for grain influences economic feasibility. Total cost per unit volume first decreases and then increases as vertical wall height increases. For a given wall height and length, cost per unit volume decreases as facility width approaches length.

INTRODUCTION

The term "flat storage" generally refers to rectangular-shaped structures with relatively low height-to-width ratios. Flat storage systems are often multipurpose structures that utilize permanent floors capable of supporting the weight of a truck or tractor. Generally, portable conveying equipment is used for filling and emptying these types of facilities, and duct-type aeration systems are employed, most often above the floor. Usually, grain is not placed directly against the wall of the building but against a portable structure indented from the edge of the facility. At the farm level, flat storage systems are often viewed as being "temporary," "stop gap," or "last choice" storage measures rather than as permanent "first choice" types of facilities. However, flat storage is often used commercially by utilizing a relatively large floor area with the grain being covered by plastic rather than a conventional roof (Siebenmorgen et al., 1986; Loewer et al., 1988). The objectives of this paper are to:

1. Present geometric design considerations for flat storage facilities.
2. Describe the influence of geometric configurations on cost of construction.

DESIGN CONSIDERATIONS

The capacity of a flat storage system (Fig. 1) may be computed by adding together the several geometric shapes that compose a pile of grain while recognizing that the grain slopes may not be uniform. Either length or width of the pile will be the limiting factor as to grain height. The following equation may be used to determine grain volume:

$$B = C * [(L * W * V) + (W^3 * \text{TAN}(t)) / 6 + ((L - W) * W^2 * \text{TAN}(t)) / 4] \dots [1]$$

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where

B = volumetric capacity for rectangular flat storage
C = conversion factor (one for SI units, 0.8 for bu if L, W and V in ft)

L = length of structure, where $L \geq W$

W = width of structure

t = angle of repose for the type of grain being stored, degrees

V = usable wall height.

The "shape" of the grain pile used in equation [1] is composed of a rectangular solid, a triangular solid and two rectangular pyramids (Fig. 1). The base formed when the two rectangular pyramids are placed together forms a square. The slope of the grain is referred to as the angle of repose. If the structure is filled uniformly, the longitudinal measurement of the base of each rectangular pyramid is one half of the width of the structure (assuming the width is less than or equal to the length). This is because the width governs the height that the grain may be placed in the structure. Therefore, the length of the triangular solid is the length of the facility minus its width. The dimensions and volumes of the different sections may be computed as follows:

$$H = \text{TAN}(t) * (W/2) \dots [2]$$

$$T = V + H \dots [3]$$

where

H = height of grain within the triangular section

T = total height of the grain pile relative to the floor
Rectangular box:

$$\text{VR} = C * L * W * V \dots [4]$$

where

VR = volume of the rectangular section

Triangular section:

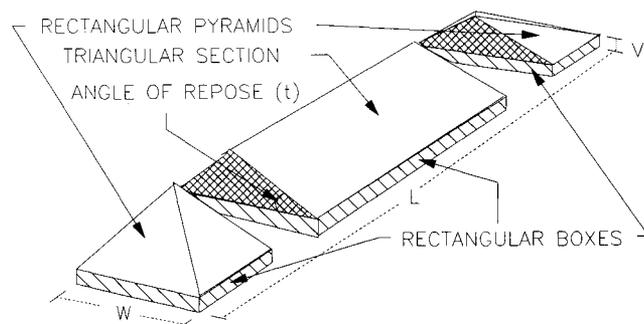


Fig. 1—Geometric shapes and flat storage.

$$VT = C * (L - W) * W * H/2 \dots\dots\dots [5]$$

where

VT = volume of triangular section
 Rectangular pyramid section:

$$VP = C * W^2 * H/6 \dots\dots\dots [6]$$

where

VP = volume of each of the pyramid sections
 Equation [1] may be derived by substituting the value for "H" in equations [5] and [6], and summing equations [4] to [6] allowing for two rectangular pyramids (Fig. 1). The incremental changes in the storage volume (B), as influenced by changes in the system length (L), width (W) and wall height (V), may be found by differentiating equation 1 with respect to L, W and V.

$$dB/dL = [W * V] + (W^2 * TAN(t))/4 * C \dots\dots\dots [7]$$

$$dB/dW = [(L * V) + (TAN(t) * (2 * L * W - W^2))/4] * C \dots\dots\dots [8]$$

$$dB/dV = [L * W] * C \dots\dots\dots [9]$$

where

$$W \leq L$$

$dB/dL, dB/dW, dB/dV$ = the changes in storage volume per unit change in structural length, width and wall height, respectively, as measured in the same units as L, W and V with $C=1$ ($C=0.8$ for obtaining changes in bushel capacity when L, W and V are measured in ft).

Equations [7] to [9] allow for the examination of the effects of changing "L", "W", "V" on total grain volume. The unit change in volume per unit change in length (dB/dL) for a given flat storage system depends on its initial width, wall height and grain angle of repose with changes becoming more pronounced as system width increases (equation [7] and Fig. 2). The unit change in volume per unit change in width (dB/dW) is also influenced by the initial length of the system with the effects of width becoming relatively less pronounced as it increases (equation [8] and Fig. 3). Unit volume change per unit increase in wall height (dB/dV) increases linearly in proportion to the initial system values of length and width (equation [9] and Fig. 4).

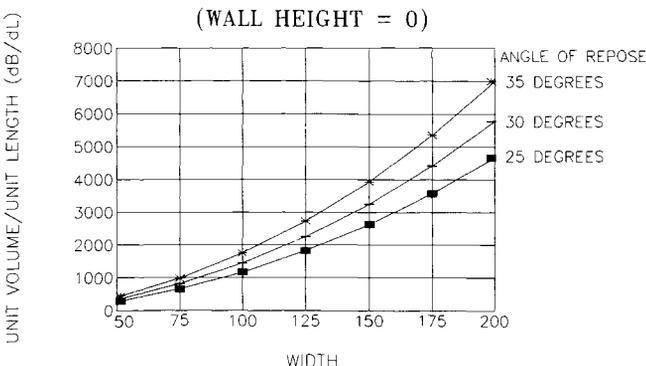


Fig. 2—Change in volume per unit length.

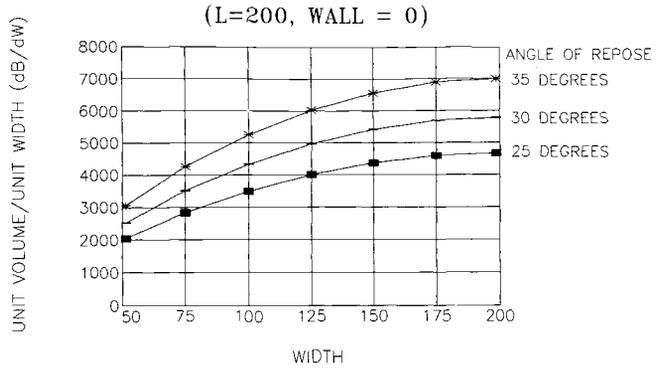


Fig. 3—Change in volume per unit width.

An important consideration in selection of facility dimensions is that the optimum storage volume to floor surface area ratio is obtained when a square floor area is used. This may be verified mathematically by ignoring "C" and substituting " $L=A/W$ " in equation [1] ("A" is floor area), differentiating with respect to "W" and solving for the optimum condition (i.e. where $dB/dW=0$). However, in most flat storage situations, width is less than length. The extent to which the volume to surface area ratio is influenced may be determined by rearranging equation [1] and dividing both sides by the floor area " $L*W$ " giving the following expression:

$$F = V + (TAN(t))*W^2/(6L) + (TAN(t))*W/4 - (TAN(t))*W^2/(4L) \dots\dots\dots [10]$$

where

F = volume to surface area ratio, $B/(L*W)$
 Taking partial derivatives of equation [10] gives the following:

$$dF/dL = (TAN(t))*W^2/(12L^2) \dots\dots\dots [11]$$

$$dF/dW = (TAN(t))*((1/4) - (W/6L)) \dots\dots\dots [12]$$

$$dF/dV = 1 \dots\dots\dots [13]$$

$$dF/dt = (SEC^2(t))*(3W*L - W^2)/(12L) \dots\dots\dots [14]$$

Equation [11] (Fig. 5) shows that the unit change in volume to floor surface area ratio per unit of length (dF/dL) decreases with facility length and increases with

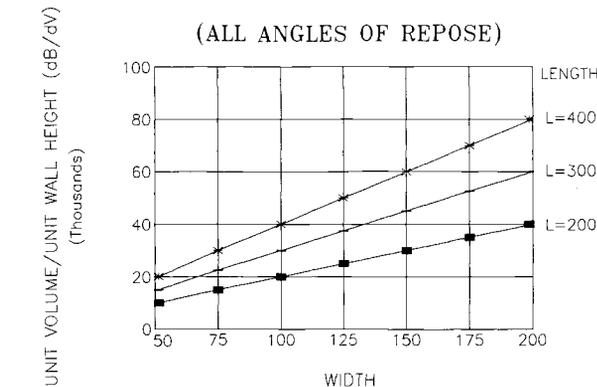


Fig. 4—Change in volume per unit wall height.

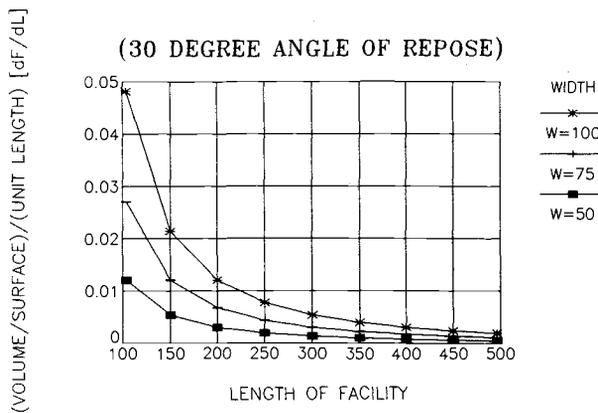


Fig. 5—Volume to surface ratio as affected by a unit change in length.

width and the grain angle of repose. Equation 12 (Fig. 6) indicates that the unit change in volume to floor surface area ratio per unit increase in width (dF/dW) increases with the grain angle of repose and length and decreases with width and reaches a maximum when "W" is equal to "L" (by definition "W" must be no greater than "L"). Equation [13] indicates that a one-unit increase in wall height will also increase the volume to floor area ratio by one, which is by far the largest of the responses. The unit change in volume to floor surface area ratio per unit increase in grain angle of repose (dF/dt) increases with grain angle of repose and initial system length (equation [14]). It increases and then decreases as initial facility width increases (Fig. 7).

RETAINING WALLS AND ECONOMIC CONSIDERATIONS

Increases in wall height result in the greatest increase in the volume to floor area ratio (equation [13]). From an economic view, however, the optimum design is dependent on the whether the cost per unit volume of "going up" is less than the cost of "spreading out." In addition, the "going up" cost usually increases disproportionately with height because of the added wall loads.

The least cost flat storage system per unit of volume is dependent on the value of base area. As land area approaches zero value, the optimum design has a square base with no retaining wall. Frequently, however, there are physical limits as to spatial expansion that, in essence, represent an economic barrier. For example, land ownership or building code restrictions may limit

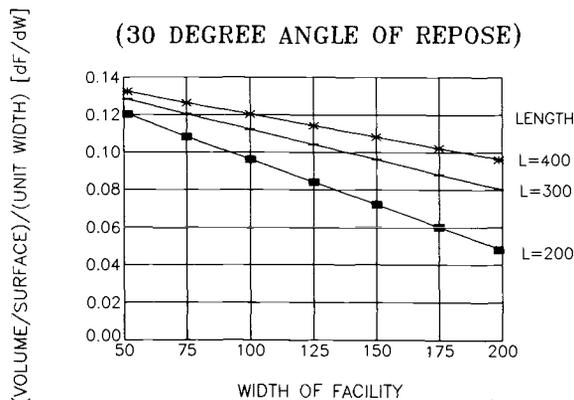


Fig. 6—Volume to surface ratio as affected by a unit change in width.

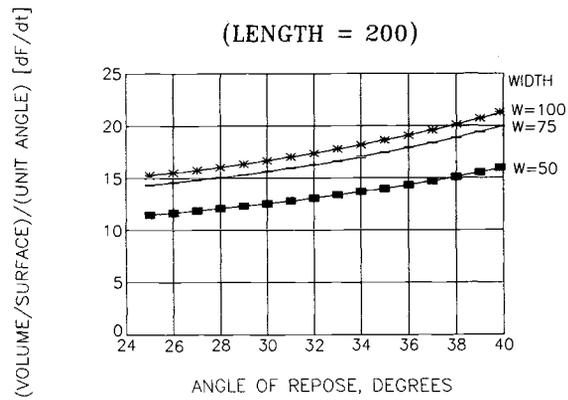


Fig. 7—Volume to surface ratio as affected by grain angle of repose.

facility dimensions. Similarly, if an existing building is to be used, then the outer boundaries of the facility are already defined. In addition, grain handling equipment or aeration capability may place limits on length, width, or height of the grain mass. When facility width is the limiting factor, additional capacity may be obtained by increasing the height of grain on the retaining wall (the retaining wall may be the outside wall of the facility or an internal wall). The force of grain against a wall increases disproportionately with wall height. Thus, the cost of the wall will also increase disproportionately. The economically optimum wall height is reached when one of the following conditions is first met:

1. design capacity is satisfied
2. maximum allowable filling height is reached
3. the marginal cost of increasing retaining wall height exceeds the cost of other storage alternatives.

Force Considerations

The lateral static wall pressure for grain in shallow bins is

$$L_s = w * Y * k \dots \dots \dots [15]$$

where

- L_s = lateral static wall pressure
- w = bulk density of grain,
- Y = depth of grain
- k = lateral-to-vertical pressure ratio.

The value of k varies with the angle of filling, the angle of internal friction, the wall angle and the coefficient of friction between the wall and the grain mass (Midwest Plan Service, 1983). Assuming vertical walls and neglecting wall friction, the generalized equation for k reduces to

$$k = \frac{\cos^2(e)}{(1 + (\sin(e) * \sin(e-a) / \cos(a))^{0.5})^2} \dots \dots \dots [16]$$

where

- e = emptying angle of repose (from horizontal)
- a = filling angle of repose (from horizontal).

With these assumptions, the value of k is constant for a specified grain and lateral pressure increases linearly with depth so that the force distribution on the walls is triangular.

If grain is piled to the maximum height along a

vertical wall of height V, the total lateral force, F, on a unit length of the wall is

$$F_L = \frac{k \cdot w \cdot V^2}{2} \dots \dots \dots [17]$$

If the wall is assumed to resist the forces as a cantilever beam with a rectangular, uniform cross section, then the maximum moment, M_m , occurs at the base:

$$M_m = \frac{k \cdot w \cdot V^3}{6} \dots \dots \dots [18]$$

The allowable stress on extreme fibers, f, of the beam is

$$f = M_m / S \dots \dots \dots [19]$$

where

S = the section modulus per unit wall length, and

$$S = d^2 / 6 \dots \dots \dots [20]$$

where

d = the wall thickness.

The amount of material in the wall, and presumably the cost in this idealized situation, is proportional to the area of the vertical cross section, or

$$A = V \cdot d \dots \dots \dots [21]$$

By rearrangement and substitution of the previous four equations into equation [21], "d" can be eliminated from the expression

$$A = (k \cdot w / f)^{0.5} V^{(5/2)} \dots \dots \dots [22]$$

Thus, in the idealized case, the cost of a cantilever wall for resisting lateral grain forces is proportional to the (5/2) power of maximum grain depth.

Structural Cost Considerations

For a more practical estimate of cost versus wall height, the actual construction costs of 30.48 m (100-ft) lengths of movable grain storage walls of four heights were estimated from plans prepared by the Midwest Plan Service (1986). Overturning moments due to lateral grain forces are resisted by transmitting vertical forces of the grain on floor panels to the adjoining sidewalls by diagonal tie-rods. The sidewalls and floor panels consist of construction-grade plywood nailed to studs and joists, with provisions for distributing forces from the tie-rods. Prices for lumber, steel and hardware were obtained from local vendors, and estimates of construction time were multiplied by local hourly rates for welders, carpenters and general laborers. Cost estimates did not allow for profits by contractors, but the totals were increased by 10% to allow for waste and the extra cost of purchasing materials in standard lots. The estimated costs are shown in Table 1. The cost per unit wall length versus wall height is shown graphically in Fig. 8 and represents the equations:

SOURCE OF EXPENSE	WALL HEIGHT, m(ft)			
	1.83(6)	2.44(8)	3.05(10)	3.66(12)
steel and hardware	220.15	248.26	371.75	505.51
lumber and plywood	934.33	1191.83	1653.00	2216.67
tie rods @ \$28 each	700.00	700.00	700.00	700.00
carpentry and assembly	270.00	306.00	360.00	396.00
TOTAL	2124.48	2446.09	3084.75	3818.17
TOTAL + 10%	2336.93	2690.70	3393.22	4199.99
Per unit length per meter	76.67	88.28	111.33	137.79
per ft	23.37	26.91	33.93	42.00

$$CWL, \$/m = 82.7376 \cdot V - 47.4438 \cdot V^2 + 18.3846 \cdot V^{(5.2)}$$

$$CWL, \$/ft = 7.6866 \cdot V - 1.3436 \cdot V^2 + 0.28741 \cdot V^{(5/2)} \dots \dots \dots [23]$$

where

CWL = cost, \$ per unit of wall length; and
V = wall height up to 3.66 m (12 ft).

This least-squares regression model crosses the origin, implying that the cost of "no wall" is zero. For short walls, costs should be nearly proportional to height because standard lumber dimensions and spacings are more than adequate for the grain loads. As wall height increases in the range of 1.8 to 3 m (6 to 10 ft), more of the wall components are loaded to near their design capacity, providing more economy relative to height. With greater heights, the load-bearing components contribute a greater proportion of the total cost and can be designed to nearly match their required capacity. The third term in equation [23] has V to the 5/2 power to estimate the disproportionate increase of materials required for higher walls (shown by equation [22]).

Economic Comparisons

Economic comparisons are dependent, in part, on establishing a "base line" among various alternative systems. The base line conditions may not be applicable to all situations, and, as with any economic comparison, changes in relative and absolute prices tend to make absolute price determinations obsolete rather quickly. Given these conditions, for this analysis it will be assumed that

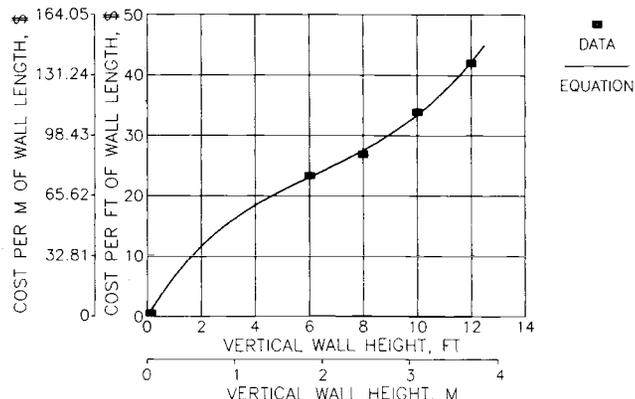


Fig. 8—Vertical wall cost.

1. Lengths and widths of the floor are predetermined.
2. Factors to be considered in computing the per-volume cost of the storage facility are the vertical wall, concrete floor, and plastic cover for the grain.
3. Only purchase prices will be compared in that other economic factors (life of the facility, interest rates, alternative uses of the structure, etc.) will be considered the same for all systems.
4. Additional costs that might be associated with corners of the floor will not be considered.

Given the volume (equation [1]), perimeter (equation [24]), floor area (equation [26]), and top surface area (equation [27]) of a facility with a specified length, width, wall height and grain angle of repose, the cost per unit volume may be determined.

$$P = 2 * L * W \dots\dots\dots [24]$$

where

- P = floor perimeter
- L = floor length (L =>W)
- W = floor width.

$$A_{\text{floor}} = L * W \dots\dots\dots [25]$$

where

A_{floor} = floor area of facility

$$A_{\text{surface}} = (W^2 / \cos t) + (W / \cos t) * (L - W) + E * P \dots\dots\dots [25]$$

Where

- A_{surface} = surface area above vertical wall
- t = grain angle of repose
- E = width of grain cover in excess of that required to cover the surface area above the vertical wall.

The costs of the floor and grain cover may be estimated by multiplying the areas of each by the respective cost per unit area of material. The total purchase cost of the facility may be obtained by adding the costs of the floor, grain cover, and vertical wall. Division of total cost by equation [1] gives total purchase cost per unit volume of grain storage capacity. The following examples illustrate the influence of design on the economic desirability of flat storage systems.

Flat Storage in Existing Structures

In situations where flat storage structures are also used for other purposes, such as a farm shop, the cost of adding a vertical wall to contain the grain is the major additional expense associated with grain storage. That is, the basic structure already exists, most often with walls that are not designed to carry the loads associated with grain. When considering only the additional cost of the vertical wall (Fig. 9), there are three economic "zones" to consider. Zone 1 is for the vertical wall height ranging from 0 to 1.2 m (0 to 4 ft), Zone 2 is from 1.2 to 2.4 m (4 to 8 ft) and Zone 3 is above 2.4 m (8 ft). The cost per bushel increases when a vertical wall is added if there are no other cost considerations such as floor and grain cover. This cost continues to increase throughout Zone 1,

stays the nearly same or decreases in Zone 2 and increases exponentially in Zone 3. The following conclusions can be drawn with regard to flat storage in existing multi-use structures:

1. The cost of adding a vertical wall cannot be economically justified if there is sufficient volume within the facility to hold the grain and the amount of space saved does not have higher alternative value.
2. On a per-bushel basis, vertical wall cost decreases or stays essentially the same for wall heights in the range between 1.2 and 2.4 m (4 and 8 ft) (Zone 2).

Temporary Flat Storage Structures

Temporary flat storage structures are defined for purposes of this discussion as being used only for grain

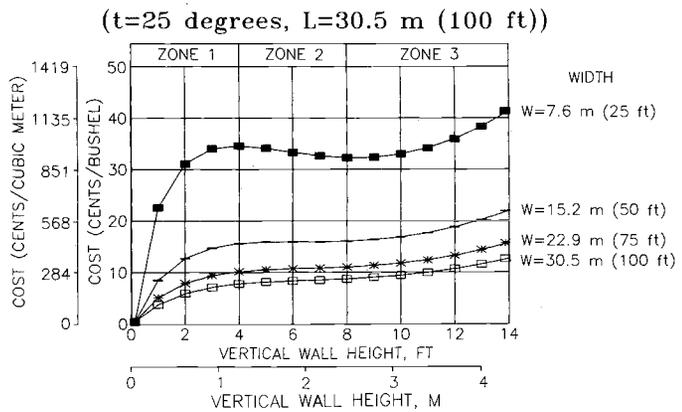


Fig. 9—Vertical wall cost per unit volume.

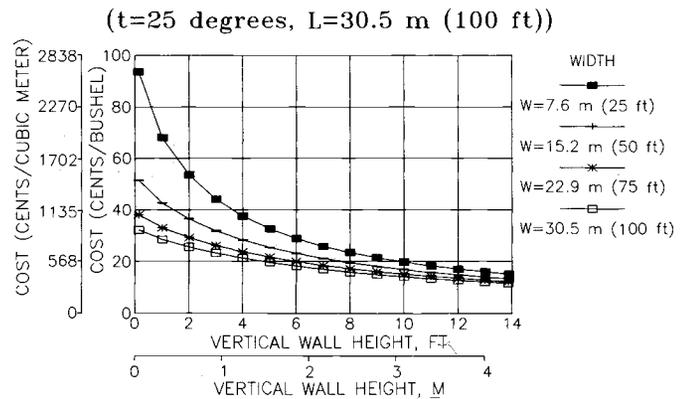


Fig. 10—Concrete floor cost per unit volume.

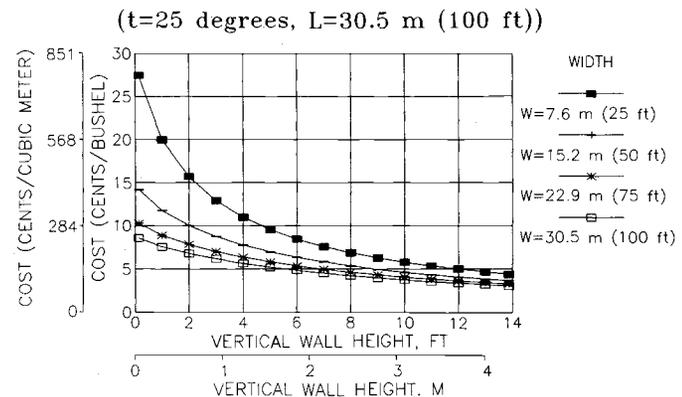


Fig. 11—Cover cost per unit volume.

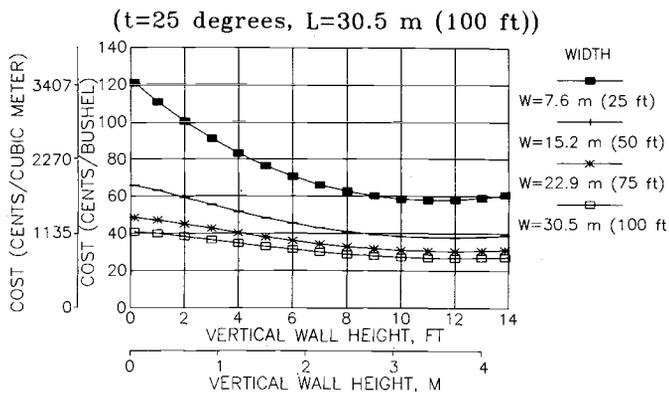


Fig. 12—Floor, cover and wall cost per unit volume of grain.

storage. The structures consist of a floor, plastic grain cover, and optional vertical wall. In Fig. 10, the cost of a concrete floor is shown using a cost of \$21.53/m² (\$2.00/ft²). Similarly, Fig. 11 shows the cost of a high quality PVC covering priced at \$4.84/m² (\$0.45/ft²) and allowing 0.61 m (2 ft) of material to extend past the perimeter of the facility. Both these costs decrease exponentially per unit volume of grain as vertical wall height increases. Total wall, floor and cover costs (Fig. 12) continue to decrease per unit of capacity until vertical wall height exceeds approximately 3.4 m (11 ft).

Proportion of costs is shown in Fig. 13 for one particular length and width.

The cost for a relatively large, temporary, flat storage structure is given in Fig. 14. The capacity of this system makes it more suited to commercial needs, and the per unit volume cost is significantly less than the smaller structure in Fig. 13.

Angle of repose of the grain is also a consideration. The greater the angle, the lower the per unit volume cost of the facility (Fig. 15). However, the differences become less as vertical wall height increases.

SUMMARY

In situations where temporary flat storage systems are to be built, the following considerations apply—given the range of prices and vertical wall heights used in this study. Again, only the costs of the vertical wall, floor, and grain cover were considered, and no allowance was made for the influence of structure shape on filling and unloading equipment.

1. Cost per unit volume continuously decreases as vertical wall height increases.
2. The costs for the floor and grain cover will usually exceed that of the vertical walls.
3. Increasing wall height decreases the floor area and maximizes the marginal increases in

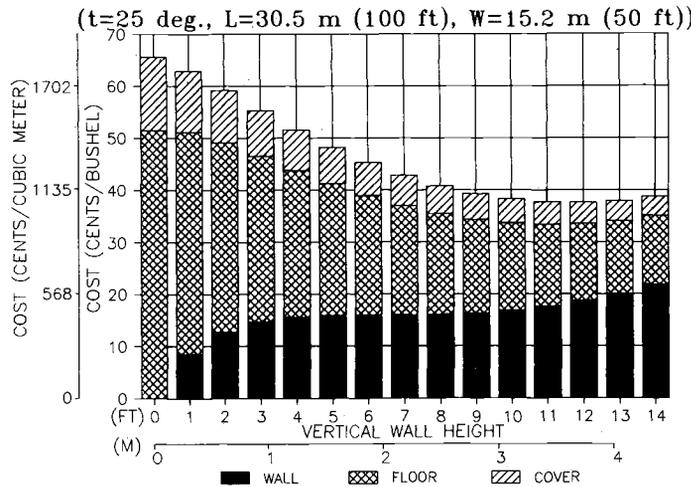


Fig. 13—Flat storage cost distribution facility 1.

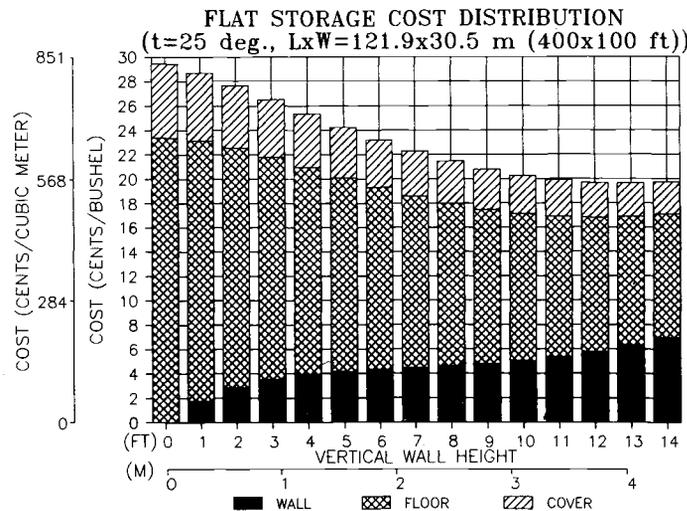


Fig. 14—Flat storage cost distribution facility 2.

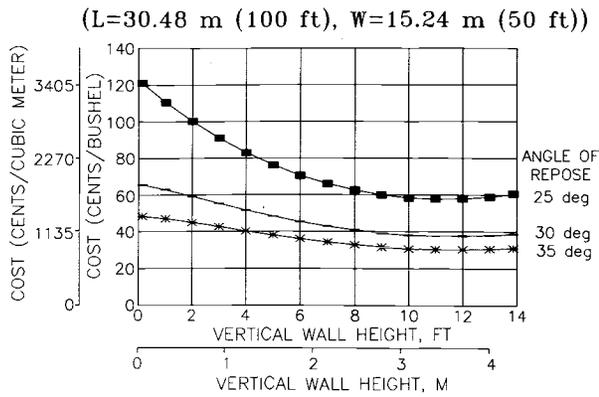


Fig. 15—Floor, cover and wall flat storage cost as affected by angle of repose.

capacity (equation [13]). Even in Zone 3 where the cost of the wall increases exponentially, the cost per unit volume continues to drop over the range tested, in part because the cost savings from the floor may be applied to the vertical wall.

4. Cost per unit volume generally decreases with increases in floor area; that is, the bigger the facility, the lower the per unit volume cost.
5. For the given wall height and length, cost per unit volume decreases as width approaches length. For a given width and wall height, cost

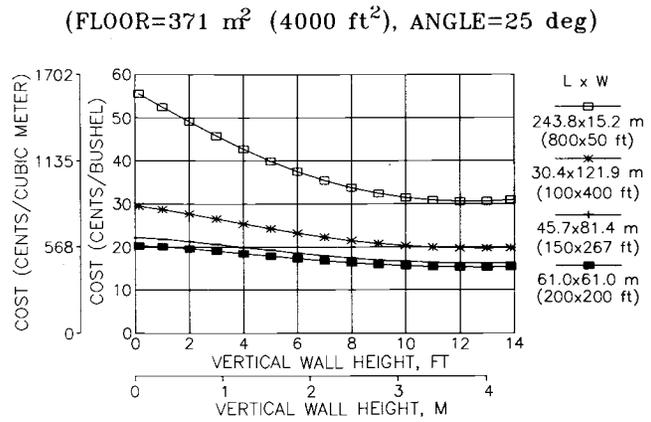


Fig. 16—Floor, cover and wall flat storage cost as affected by base dimension.

per unit volume decreases as length increases. However, the lowest cost per unit volume is for a square floor area (Fig. 16).

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