MATHEMATICAL MODELING OF A CROSS-FLOW RICE DRYER

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ABSTRACT. Cross-flow dryers are the most popular industrial-scale rice dryers used in the U.S., yet few mathematical models have been developed and rigorously validated for such dryers. In addition, the glass transition states of rice kernels have never been predicted using a deep-bed drying model. In this study, a mathematical model was developed that describes the distribution of grain and air properties throughout a cross-flow dryer column. The model was validated by performing experiments in a lab dryer that was fabricated to simulate cross-flow drying. The model predictions of grain and air properties were observed to be very close to the measured values in the drying experiments; the root mean square error between the predicted and measured values of rice MC, air temperature, and air RH were less than one percentage point, 5°C, and ten percentage points, respectively. The model was then used to predict the glass transition state of starch present in rice kernels throughout the dryer column. The impact of initial MC on the glass transition states of rice kernels during cross-flow drying was also illustrated. Such predictions of rice kernel material states allow the model to be used for rice fissuring research. Additionally, the model could be applied to optimize drying operation parameters as well as improve dryer design, so as to achieve greater drying capacity, milling quality, and energy efficiency in a commercial drying operation.

Keywords. Cross-flow drying, Glass transition, Mathematical model, Rice drying.
ing four properties of the grain and air: grain MC, grain temperature, air humidity, and air temperature. While the grain MC and air temperatures across the grain bed have been validated in a few models (Morey and Li, 1984; Noomhorm and Verma, 1986), the majority of model validations have been performed by comparing only the average grain MC. A comprehensive validation of such models requires comparison of the aforementioned four properties across the entire grain bed, which was not found in the literature. Such validation becomes even more important for cross-flow drying, where kernels dry non-uniformly across the column thickness. Despite the widespread industrial application, only a few models have been reported that describe cross-flow dryers (Rumsey and Rovedo, 2001).

In contrast to other major grains, such as corn and wheat, that are primarily consumed in flour form, the majority of rice is consumed as intact kernels, making it critical to retain kernel integrity during processing. The presence of cracks or fissures in rice kernels is one of the most important defects affecting the economic value of rice. In milling, such kernels typically fracture at the fissure site and produce broken kernels.

Fissuring in rice kernels during the drying process has been hypothesized to be based on the properties of rice starch (Cnossen and Siebenmorgen, 2000). During drying, particularly high-temperature drying, rice kernels become heated; if a kernel is heated above its glass transition temperature, its starch undergoes a transition from a glassy to a rubbery state. Compared to the glassy state, starch in the rubbery state has greater expansion coefficients, specific volume, moisture diffusivity, and thermal conductivity (Perdon et al., 2000; Cnossen et al., 2002; Yang et al., 2003b). Intra-kernel transition of rice starch from a rubbery to a glassy state was identified to influence fissuring in rice kernels during drying and tempering (Cnossen and Siebenmorgen, 2000; Schulterman and Siebenmorgen, 2007).

Rice kernels experience non-uniform drying in cross-flow dryers because the properties of the grain and air vary across the thickness and height of the dryer (Schulterman and Siebenmorgen, 2004). During their passage through the column, some kernels may also undergo a change in their starch state from glassy to rubbery, or vice-versa. Because the material state of starch within a rice kernel depends on its MC and temperature, the state of rice kernels at each location in the dryer could be predicted with a mathematical model. Such predictions could improve the understanding of the fissuring phenomenon during commercial rice drying (Yang et al., 2003a). Furthermore, these findings could be used for development of optimum drying conditions that minimize kernel fissuring and ultimately improve the economic value of rice. None of the models describing cross-flow dryers have investigated such changes in material states of rice kernels in the dryer column.

The goal of this study was to develop a mathematical model that accurately describes the heat and mass transfer processes between rice kernels and air in a cross-flow dryer. The model was validated with experimentally measured values of rice MC, air relative humidity, and air temperature across the bed thickness. The model was then employed to predict the material state of rice kernels throughout a cross-flow dryer.

**MODEL DEVELOPMENT**

A set of four non-linear partial differential equations (PDEs) has been used in most non-equilibrium drying models. These PDEs were derived by performing heat and mass balances between the grain and drying air in a control volume inside a dryer (Brooker et al., 1992). Numerical integration techniques are used to solve the resulting system of PDEs. An alternate approach is taken in the model presented in this study, wherein the heat and mass balances are directly applied to discretized volume elements (voxels) in a cross-flow dryer. It is proposed that such an approach is simpler to
comprehend and develop, as it avoids the formulation of PDEs using differentiation, and later their integration.

**MODEL GEOMETRY**

In the presented model, the entire grain column can be conceptualized to comprise numerous voxels that are rectilinearly arranged and are fixed relative to the dryer (fig. 2). The grain and air travel through these voxels during the drying process. When the volume of each voxel is extremely small, the properties of the grain and air contained in any given voxel can be assumed to be uniform throughout the voxel. As long as the flow rates and properties of the incoming grain and plenum air remain constant, the cross-flow dryer will be in steady state, i.e., the properties of the grain and air in any voxel remain unchanged.

Rice kernels are assumed to fall vertically along the z-axis, while the air flows solely in the thickness direction of the column (x-axis in fig. 2). Because all the voxels that are located along the width of the column (y-axis in fig. 2) encounter air and kernels with identical properties, these voxels experience identical drying conditions. Therefore, these voxels could be represented by just one voxel. With this simplification, cross-flow drying becomes a two-dimensional steady-state phenomenon.

The three dimensions of each voxel are represented as $\Delta x$, $\Delta y$, and $\Delta z$; their magnitudes are typically selected based on the available computation power and the desired accuracy of model predictions. Assuming the grain speed to be $v_g$ (m s$^{-1}$), the duration for grain to travel from one voxel to the next voxel position will be $\Delta t = \Delta z / v_g$. It should be noted that the grain and air spend different durations in a voxel, which is determined by the voxel dimensions and the respective flow rates.

**ASSUMPTIONS**

The following assumptions were made in the development of the presented model:

1. Shrinkage of kernel volume is negligible during the cross-flow drying process.
2. Intra-kernel temperature gradients are negligible.
3. Conductive heat transfer between kernels is negligible.
4. The plenum air temperature and humidity are uniform at the inlet to the grain column.

These assumptions are routinely made in non-equilibrium grain drying models (Brooker et al., 1992; Srivastava and John, 2002).

**COMPUTATION SCHEME**

In the presented model, the conditions of the grain and air in any voxel of the cross-flow dryer are described by four properties: MC of grain ($M$, decimal, d.b.), humidity ratio of air ($W$, kg water vapor kg$^{-1}$ dry air), temperature of grain ($\theta$, $\degree$C), and temperature of air ($T$, $\degree$C). The properties of the grain and air throughout a voxel are assumed to be the same values at which they entered the voxel. For example, the layer of voxels at the top of the grain column will have the same grain properties throughout the voxels as in the garner bin. Similarly, the voxels located adjacent to the heated-air plenum will have identical air properties throughout the voxels as in the dryer plenum.

A series of heat and mass balance equations are solved in each voxel to determine the properties of the grain and air leaving the voxel. To solve these equations, knowledge of the inlet conditions of the grain and air in a voxel is required. Therefore, a specific sequence of voxel computations was employed in this model, which ensures that the inlet properties are known for every voxel from earlier steps. In the example shown in figure 3, the equations for voxel 10 require the inlet properties of grain and air, which were determined earlier from the computations for voxels 9 and 3, respectively.

**MODEL EQUATIONS IN A VOXEL**

The model equations described in this section predict the grain and air properties as they are exiting the voxel; these properties are represented by $M$, $W$, $\theta$, and $T$. For each voxel,
the inlet grain and air properties are represented with the subscript \( i \) and are \( M_i, W_i, \theta_i \), and \( T_i \).

First, the relative humidity of the incoming air (RH, decimal) was evaluated using psychrometric relationships. Next, the equilibrium MC of rice (\( M_e \), decimal, d.b.) was calculated at the voxel inlet air conditions, i.e., \( T_i \) and RH, using the modified Chung-Pfost equation (table 1). When the computed value of RH, is greater than 0.99, the logarithmic nature of the Chung-Pfost equation predicts unrealistically high \( M_e \). In such situations, \( M_e \) is determined assuming RH to be 0.99. The following thin-layer drying equation was then applied to determine \( M \):

\[
\frac{M - M_e}{M_i - M_e} = e^{-k\Delta t} \tag{1}
\]

where \( k \) is an experimentally determined drying constant, which depends on the temperature of the drying air (\( T_i \)). The thin-layer drying experiments reported by Cnossen et al. (2002) were used to evaluate \( k \) at different air temperatures in the range of 40°C to 60°C, and then a linear regression equation was developed to describe the temperature dependence of \( k \) (table 1). In place of equation 1, Page’s equation that utilizes two parameters to describe thin-layer drying could also be used (Page, 1949). However, the use of two parameters makes it difficult to accurately describe the temperature dependence of the Page equation parameters. ASABE Standard S448.2 (ASABE, 2014) describes a set of temperature-dependent Page parameters for short-grain rice, but these correlations were deemed to be non-applicable for long-grain rice, which dries more rapidly (Prakash and Pan, 2012).

The amount of moisture lost by kernels in a voxel, when its MC changes from \( M_i \) to \( M \), can be expressed as \( \Delta m_g \) (kg):

\[
\Delta m_g = \Delta t \Delta y \Delta z \frac{\rho_g}{(1 + M_i)} (M_i - M) \tag{2}
\]

where \( \rho_g \) (kg m\(^{-3}\)) is the bulk density of grain, which depends on the MC of grain; \( \rho_g \) was assumed to be constant for grain in a voxel and was calculated at the inlet grain MC (\( M_i \)). Similarly, the amount of moisture gained by the air flowing through the voxel in \( \Delta t \) duration can be described by:

\[
\Delta m_a = v_a \Delta t \Delta y \Delta z \rho_{da} (W - W_i) \tag{3}
\]

where \( v_a \) is the superficial velocity of dry air (m s\(^{-1}\)), and \( \rho_{da} \) is the density of dry air (kg m\(^{-3}\)), which depends on \( T_i \) and \( W_i \). When the mass flow rate of air (\( G_{ai} \), kg s\(^{-1}\) m\(^{-2}\)) is known, the value of \( v_a \) can be given by \( v_a = G_{ai} / \rho_{da} \) (Brooker et al., 1992).

In any given time interval, all moisture that is lost by the grain in a voxel becomes part of the air flowing through the voxel, and therefore \( \Delta m_g = \Delta m_a \). Equating and rearranging equations 2 and 3, \( W \) can be calculated by:

\[
W = W_i + \frac{\rho_g}{\rho_{da}} \frac{\Delta x}{v_a \Delta t} (M_i - M) (1 + M_i) \tag{4}
\]

Instantaneous heat transfer from the air to the grain in a voxel (\( q_{\text{in}}, W \)) is given by:

\[
q_{\text{in}} = h(\alpha_i \Delta t \Delta y \Delta z \Delta t) (T - \theta) \tag{5}
\]

where \( h \) is the convective heat transfer coefficient (W m\(^{-2}\) ºC\(^{-1}\)), and \( \alpha_i \) is the kernel surface area per unit bulk grain volume (m\(^2\) m\(^{-3}\)). Using correlations reported for convective heat transfer in packed beds of spheres, \( h \) was calculated (Geankoplis, 2000):

\[
J_H = e \left[ \frac{h}{c_p G_a} \left( \frac{c_p H}{k} \right)^{2/3} \right] = \frac{2.876}{Re} + 0.3023 \frac{0.35}{Re} \tag{6}
\]

where \( e \) is the porosity or void fraction, \( J_H \) is the Colburn J-factor, \( c_p \) is the specific heat of air (J kg\(^{-1}\) ºC\(^{-1}\)), \( \mu \) is the viscosity of air (Pa s), \( k \) is the thermal conductivity (W m\(^{-1}\) ºC\(^{-1}\)), and \( Re \) is the Reynolds number. The subscript \( f \) indicates that those properties are evaluated at the film temperature, while other air properties are evaluated at the bulk air temperature. Since individual rice kernels are assumed to be isothermal, the film temperature (\( T_f \), ºC) is determined as: \( T_f = (\theta + T_i) / 2 \).

The heat utilized for moisture vaporization at any given time (\( q_{\text{vap}}, W \)) is given by:

<table>
<thead>
<tr>
<th>Table 1. Physical, hygroscopic, and thermal properties of rough rice.</th>
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<tbody>
<tr>
<td>Property</td>
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<tr>
<td>Equilibrium moisture content (% d.b.)</td>
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<td></td>
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<tr>
<td></td>
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<tr>
<td>Drying constant (s(^{-1}))</td>
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<td>Bulk density (kg m(^{-3}))</td>
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<td>Latent heat of vaporization from</td>
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<td>long-grain rice kernels (J kg(^{-1}))</td>
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<tr>
<td>Specific heat (J kg(^{-1}) ºC(^{-1}))</td>
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<tr>
<td>Porosity (decimal, °C(^{-1}))</td>
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<tr>
<td>Glass transition temperature (°C)</td>
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</table>

\(^{[a]} T \) is air temperature (°C), RH is relative humidity of air (%), MC\(_{\text{Cw}}\) is moisture content of rough rice (% w.b.), \( \theta \) is temperature of rice kernels (°C), \( M_i \) is initial moisture content (decimal, d.b.), and \( M \) is final moisture content (decimal, d.b.).
\[ q_{\text{vap}} = \lambda \frac{\rho_g}{(1 + M_i)} \Delta x \Delta y \Delta z \frac{\partial M}{\partial t} \] (7)

where \( \lambda (\text{J kg}^{-1}) \) is the latent heat of vaporization of moisture from rice kernels at the drying air temperature. The instantaneous sensible heat accumulation in the grain in the voxel \( (q_{\text{acc}}, W) \) is given by:

\[ q_{\text{acc}} = \rho_g \Delta x \Delta y \Delta z \frac{\partial \theta}{\partial t} \] (8)

where \( \rho_g (\text{J kg}^{-1} \text{C}^{-1}) \) is the specific heat of rice. The heat balance equation for grain in the voxel can be written as:

\[ q_{\text{in}} = q_{\text{vap}} + q_{\text{acc}} \] (9)

Inserting the expressions from equation 5, 7, and 8 into equation 9 and rearranging gives:

\[ \frac{\partial \theta}{\partial t} = P(\theta - T) + Q \] (10)

where \( P = \frac{-h_{\text{ad}}}{\rho_g c_g} \) and \( Q = \frac{-\lambda}{c_g (1 + M_i)} \frac{\partial M}{\partial t} \) (11)

For a small duration \( \Delta t \), the changes in the air and grain properties are assumed to be very small. Therefore, \( P \) and \( Q \) can be considered constant for this duration. The value of \( \frac{\partial M}{\partial t} \) is estimated as \( (M_i - M) / \Delta t \), whereas other terms of \( P \) and \( Q \) are determined considering the following estimate for grain and air properties: \( (M + M_i)/2, (W + W_i)/2, \theta_i, \) and \( T_i \). With these assumptions, equation 10 becomes a first-order ordinary differential equation (ODE). Using separation of variables, an analytical solution for this ODE can be obtained:

\[ \theta = \theta_i e^{P \Delta t} + \left( \theta_i - \frac{Q}{P} \right) \left(1 - e^{P \Delta t} \right) \] (12)

In the duration \( \Delta t \), the total heat needed for evaporation \( (Q_{\text{vap}}, J) \), the total sensible heat accumulation in the grain \( (Q_{\text{acc}}, J) \), and the total heat supplied by the air \( (Q_{\text{air}}, J) \) can be expressed as:

\[ Q_{\text{vap}} = \lambda \frac{\rho_g}{(1 + M_i)} \Delta x \Delta y \Delta z (M_i - M) \] (13)

\[ Q_{\text{acc}} = \rho_g \Delta x \Delta y \Delta z \frac{\partial \theta}{\partial t} \] (14)

\[ Q_{\text{air}} = c_{\text{da}} (v_{\text{ad}} \Delta x \Delta y \Delta z) \left( c_{\text{da}} + W_i c_{\text{cv}} \right) (T_i - T) \] (15)

where \( c_{\text{da}} \) is the specific heat of dry air \( (\text{J kg}^{-1} \text{C}^{-1}) \), and \( c_{\text{cv}} \) is the specific heat of water vapor \( (\text{J kg}^{-1} \text{C}^{-1}) \). Because drying air is providing the latent heat for moisture vaporization and the sensible heat for heating grain, the heat balance equation for drying air can be written as:

\[ Q_{\text{air}} = Q_{\text{vap}} + Q_{\text{acc}} \] (16)

Inserting the expressions from equation 13, 14, and 15 into equation 16 and rearranging, \( T \) can be determined as:

\[ T = T_i - \left( \frac{\rho_g}{\rho_{\text{da}}} \right) \left( \frac{\Delta x \Delta y \Delta z}{v_{\text{ad}}} \right) \left( \frac{1}{c_{\text{da}} + W_i c_{\text{cv}}} \right) \times \left( \lambda (M_i - M) / (1 + M_i) + c_g (\theta - \theta_i) \right) \] (17)

Once the temperature and MC of rice in a voxel are known, the glass transition temperature for rice kernels is calculated using the equation in table 1. If the kernel temperature is greater than the calculated glass transition temperature, rice kernels are determined to be in the rubbery state; otherwise, the kernels are considered to be in the glassy state (Schulterman and Siebenmorgen, 2007). Table 1 also lists the expressions used for calculating the physical, hygroscopic, and thermal properties of rice. All properties of air were estimated using psychrometric relationships (ASHRAE, 2009).

The mathematical model was developed and solved in the MATLAB computation environment (ver. R2015b, The MathWorks, Inc., Natick, Mass.). For all model simulations, the voxel thickness \((\Delta t)\) was selected to be 1.27 mm, and the grain duration in a voxel \((\Delta t)\) was 30 s. The computation time was typically less than 120 s on a desktop computer (Dell, Intel i7-4790 CPU, 3.60 GHz, 16 GB memory).

The procedure presented in this model for evaluation of \( M, W, \theta, \) and \( T \) is sequential. Several alternative computation algorithms were tried to improve the model predictions; however, the added complexity did not significantly improve the accuracy of predictions. This may be attributed to the small values of \( \Delta t \) and \( \Delta t \) (or \( \Delta z \)) that were used in the simulations, which caused the changes in the grain and air properties across the voxel to become very small compared to their absolute values. In addition, the temperature and moisture dependence of various grain properties was not known to an accuracy level that would allow benefit from more efficient computational algorithms.

**Correction for Infeasible Air Properties**

The model equations are based on several assumptions and approximations, which sometimes result in infeasible air properties. For example, the computed \( W \) in a voxel may be so high that the air RH becomes larger than 100%. Such situations arise primarily due to lack of accurate information for the thin-layer drying equation and the moisture isotherm equation over the entire range of air conditions that might exist in a cross-flow dryer. When such situations arise during simulations, the RH is artificially forced to 100% and air temperature is increased so that the air can still have the computed value of \( W \) (Noomhorm and Verma, 1986). Such an override of the computed value of air temperature could be avoided by including condensation and rewetting phenomena in the model. However, since such a situation is observed in less than 1% of the voxels, the added complexity of including these phenomena was deemed unnecessary.

**Equivalence Between Cross-Flow and Stationary-Bed Dryers**

Despite differences in grain flow, the drying phenomena in cross-flow and stationary-bed dryers can be represented by the same model. The only modification required is the
replacement of the column height variable \( z \) by a drying duration variable \( t \) using the following relationship: 
\[
t = \frac{z}{v_g}
\]  
(Brooker et al., 1992). With this change, the cross-flow drying transforms from a two-dimensional steady-state to a one-dimensional transient-state phenomenon.

The conceptual equivalence between a cross-flow and stationary-bed dryer is shown in figure 4. The drying column of the cross-flow dryer is shown as figure 4a, which is just a rotated view of the grain column. Here, the position of grain is shown in terms of its distance relative to the plenum \( (x_i) \) and the time spent within the column \( (t_i) \). Figure 4b shows a schematic of a stationary-bed dryer with an identical number of voxels in the \( x \)-direction; however, the grain is static and its properties change in the time direction. Such equivalence makes it easier to validate a cross-flow dryer model with a stationary-bed dryer, which is easier to fabricate and test at the laboratory scale.

Despite the theoretical equivalence, the operating conditions vary between commercial cross-flow dryers and stationary-bed dryers, such as bin dryers. The temperature and flow rates of the drying air are generally higher in the former, which results in greater grain drying rates. In this study, the cross-flow drying model was validated by experiments performed in a laboratory-scale, stationary-bed dryer. The drying conditions were selected to match the typical operating conditions observed in a commercial cross-flow dryer.

**MATERIALS AND METHODS**

**RICE SAMPLES**

Rice of long-grain cultivar ‘Roy J’ was used in the drying experiments. The rice was harvested at the University of Arkansas Rice Research and Extension Center near Stuttgart, Arkansas, at approximately 22% MC. The harvested rice was cleaned using a dockage tester (XT4, Carter-Day, Minneapolis, Minn.) to remove foreign materials and unfilled kernels and then stored at approximately 4°C in sealed containers, prior to the drying experiments.

**LABORATORY DRYER**

A laboratory-scale dryer was fabricated to validate the dryer model. Drying air at the desired temperature and RH was produced by a 0.91 m\(^3\) controlled-environment chamber (ESPEC, Hudsonville, Mich.) using in-built electrical heating, refrigeration, and control components (fig. 5). A 0.56 kW centrifugal fan (Dayton Electric Manufacturing Co., Chicago, Ill.) was mounted outside the chamber to avoid high-temperature exposure. The fan suctioned air from the chamber and forced it through the grain in the drying bed via insulated air ducts.

The dryer assembly inside the controlled-environment chamber comprised a wooden box, a cylindrical acrylic-glass sample holder that served as the drying column, and a
set of ten fiber-mesh cylindrical woven baskets. The wooden box acted as a plenum, and the air properties therein were measured to represent the inlet air conditions to the rice column. The sample holder had a metallic screen as a base. At the bottom of the sample holder, a rubber O-ring was fitted to ensure an airtight fit with the wooden box. To divide the drying column into ten discrete layers, rice was placed in woven baskets that fit snugly into the sample holder. Use of these baskets facilitated accurate measurement of rice MC in each layer of the rice bed after a drying run was completed. Each basket was instrumented with a standalone datalogger (HOBO T/RH 2.5%, Onset Computer Corp., Bourne, Mass.) that continuously measured the temperature and RH of the drying air. The dataloggers occupied about 7% of the volume in each basket. Two dataloggers were also placed in the wooden box to record the plenum air conditions.

The grain bed in the lab dryer was stationary during experiments. However, based on the equivalence between cross-flow and stationary-bed dryers, the lab dryer could be used to study cross-flow drying of grain. The kernels in the bottom basket of the lab dryer represented the kernels located near the heated-air plenum, whereas the kernels in the top basket represented kernels located farthest from the plenum of a cross-flow dryer. The unique design of the lab dryer facilitated measurements of grain and air properties at ten locations across the grain bed, which made it possible to quantify the non-uniformity of cross-flow drying at an unprecedented resolution.

**Drying Experiments**

Rice was taken from the refrigerated storage and equilibrated to room temperature for 24 h prior to drying runs. A 270 g sample of rice was placed in each of the ten baskets, and the baskets were vertically arranged in the sample holder. In such a configuration, the shape of the resulting rice bed was cylindrical with a diameter of 127 mm and height of 380 mm.

Table 2 lists the initial MC and temperature of the rice and the temperature and RH of the inlet (or plenum) air for each drying run. The inlet air conditions varied very little during any single drying run. The MC of rice was determined by drying duplicate 15 g subsamples in a convection oven, which was operated at 130°C for 24 h (Jindal and Siebenmorgen, 1987). The experimental setup allowed continuous measurement of air temperature and RH at ten locations in the grain column. However, the grain MC could be measured only once, at the end of a drying run. Therefore, three drying runs, each with a different drying duration, were performed, which enabled quantification of MC profiles for three different durations. During the drying runs, the plenum air temperature was observed to be slightly less than the 60°C temperature that was set in the controlled-environment chamber. Such reduction in air temperature could be attributed to heat loss to the ambient surroundings through the air ducts.

Once drying for the desired duration was completed, a sample of rice was taken from each basket and MC was measured using the oven method mentioned earlier. The air temperature and RH at each datalogger location were recorded at 1 min intervals. For all drying runs, the airflow rate through the rice was 0.56 m³ s⁻¹ m⁻², which was measured with a vane-type anemometer (HHF141, Omega Engineering, Inc., Norwalk, Conn.).

For each of the three drying runs, a simulation was performed, and the root mean square error (RMSE) was determined to quantify the degree to which the model predicted the experimental data:

$$\text{RMSE} = \sqrt{\frac{\sum_{n=1}^{N} (X_m - X_e)^2}{N}}$$

where \(X\) is the grain or air property being compared, and \(N\) is the number of compared data points. The subscripts \(m\) and \(e\) indicate whether the property was determined from the model or experiments. Because experimentally determined values of grain MC, air temperature, and air RH were available for ten locations in the rice column, the value of \(N\) was 10.

The small size of a rice kernel makes it extremely difficult to accurately measure grain temperature, especially in a deep-bed dryer; therefore, grain temperature was not measured in this study. The inability to validate grain temperature has been a perennial limitation in the rice drying modeling literature. Despite this limitation, the present study provides a more comprehensive validation for a deep-bed rice drying model than previously reported. Grain MC, air temperature, and air RH were validated in the presented model, across the column thickness as well as at several drying durations.

**RESULTS AND DISCUSSION**

**Model Validation**

Experimental and simulated rice MC profiles for the three drying runs are compared in figure 6. As expected, rice kernels located near the heated-air plenum dried faster than those located farther from the plenum. The difference in measured MC between the two sides of the drying column was 3.6 percentage points after 30 min of drying. Similar non-uniform drying of rice kernels across the column thickness was also observed in a commercial cross-flow dryer (Schulterman and Siebenmorgen, 2004).

The model predictions follow the experimental MC results closely, with the difference between being less than one percentage point in most cases (fig. 6). Rumsey and Rovedo (2001) and Zare et al. (2006) validated their models by comparing average grain bed MC; the errors in their model predictions were similar to those reported in this study. However, accurate prediction of average grain bed MC alone is not sufficient to validate a deep-bed drying model. Few studies have reported model validation by comparing the MC profiles across the grain bed. Noomhorm and Verma (1986) measured grain and air properties at several locations in a

| Table 2. Grain and inlet air conditions for the three drying runs.[a] |
|------------------|--------|--------|--------|
|                  | Run 1  | Run 2  | Run 3  |
| Drying duration (min) | 30     | 60     | 90     |
| Initial rice MC (%)   | 20.5   | 20.5   | 20.5   |
| Initial air temperature (°C) | 21.5   | 19.6   | 22.7   |
| Inlet air temperature (°C) | 56.6 (0.3) | 57.3 (0.5) | 58.5 (0.4) |
| Inlet air RH (%)      | 13 (1) | 13 (1) | 11 (1) |

[a] Values in parentheses are standard deviations of measured values.
ported this statistic. When Newton and Page equations were applied to rice, none of the deep-bed drying models have reported this statistic. When Newton and Page equations were applied to rice, none of the deep-bed drying models have been frequently reported in thin-layer drying studies for rough rice, which is approximately equivalent to an RMSE value of 1.6 percentage points. While RMSE values have typically varied in the 4 to 7 and 0.3 to 0.8 percentage point ranges, respectively (Hacihafizoglu et al., 2008).

Discrepancies between model predictions and experimental measurements could be attributed to four factors: lack of accurate grain properties, assumptions made in the model development, accuracy of instruments, and biological variations in rice samples. Among the listed factors, the most important are inaccuracies in grain properties, particularly, the grain EMC and drying constant, which are determined by moisture isotherm and thin-layer drying equations, respectively (Brooker et al., 1992). Even when grain EMC and drying constants are available for long-grain rice, they are rarely reported for the entire range of temperature and RH conditions that rice kernels may encounter in a cross-flow dryer. Furthermore, the temperature and RH dependence of the drying constants are yet to be accurately expressed in an equation form. In addition, convective heat transfer coefficients have not been determined for rice kernels. Since the shape of an object is known to affect the convective heat transfer coefficient, the use of correlations that were developed for other shapes, such as spheres, could underestimate or overpredict the heat transfer in the model. Additionally, the use of smaller T/RH dataloggers in the lab dryer would have been less intrusive and could have further improved model validation results.

Glass Transition States of Rice Kernels

The glass transition states of starch within a rice kernel were determined based on the rice temperature and MC. Assuming rice kernels to have uniform intra-kernel MC and temperature, the material state of the rice kernels in each voxel was predicted by the model. Figure 8 shows the MC, temperature, and corresponding material state of rice kernels in every voxel as they travel through the cross-flow dryer. For this simulation, the drying conditions were assumed to be the same as run 1 (table 2). In figure 8, the movement of rice kernels is vertically downward while the direction of airflow is left to right. The thickness of the drying column is 380 mm, and the time taken by rice kernels to travel the entire drying column is 30 min. The column thickness is linearly increased in scale to enhance the clarity of the visualization.

The non-uniformity of grain MC and temperature in a cross-flow dryer produces a variation in material states of rice kernels: some kernels are in a glassy state, whereas others are in a rubbery state. For the drying conditions in figure 8, all rice kernels began their passage through the column in the glassy state but eventually transitioned to the rubbery state. The spatial distributions of material states and rice temperatures are similar, suggesting that the changes in the material state are strongly correlated with the increase in rice temperature as kernels travel downward in the column. Compared to temperature, the MC of rice kernels changes more slowly and affects the material state to a smaller extent in this illustration.

Cross-flow drying is typically performed in multiple passes, with each subsequent pass having successively lower initial moisture content (IMC) of the rice. When the simulation was run for different IMCs, there were stark differences in the kernel material state distribution throughout the dryer (fig. 9). For rice with an IMC of 20%, all kernels left the dryer in rubbery state, while for rice with an IMC of 16%, all kernels left the dryer in glassy state. When rice IMC was 18%, the kernels near the plenum were in rubbery state.

Figure 6. Comparison of rough rice moisture content profiles within a grain column, as predicted by a mathematical model and measured experimentally, after drying for three different durations. The initial moisture content of rice in these experiments and in the corresponding simulations was 20.5%, wet basis.
while the kernels near the exhaust remained in the glassy region. For these simulations, rice kernels were assumed to spend 30 min in the drying column and the remaining drying conditions were the same as run 1 in table 2.

The preceding discussion clearly establishes that the non-uniformity of drying in cross-flow dryer produces a variation in the material states of rice kernels throughout the dryer, both vertically and horizontally. After drying, rice kernels exiting the column are mixed in a tempering bin. During tempering, rice kernels undergo further moisture exchange, which may also change their material states. Glass transition of starch is a reversible phenomenon (Biliaderis et al., 1986; Zeleznak and Hoseney, 1987); therefore, rice kernels could experience multiple transitions between glassy and rubbery states during a typical drying operation. While the relationship between material states and rice fissuring is yet to be completely understood, the presented model is a powerful research tool for investigating this relationship and thereafter applying the gained knowledge to reduce fissuring in commercial-scale rice drying.

CONCLUSION

A mathematical model was developed to predict grain and air properties in a cross-flow dryer using a novel approach in which the heat and mass balance equations were directly applied to discretized volume elements in the dryer.

<table>
<thead>
<tr>
<th>Property</th>
<th>Drying Duration (Drying Run)</th>
<th>30 min (Run 1)</th>
<th>60 min (Run 2)</th>
<th>90 min (Run 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rice MC (percentage points)</td>
<td></td>
<td>0.9</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Air temperature (°C)</td>
<td></td>
<td>3.2</td>
<td>4.7</td>
<td>4.7</td>
</tr>
<tr>
<td>Air RH (percentage points)</td>
<td></td>
<td>9.7</td>
<td>7.9</td>
<td>3.7</td>
</tr>
</tbody>
</table>
To validate the model, a lab-scale dryer was fabricated, and drying experiments were performed. Model predictions for the MC of grain, air temperature, and air relative humidity matched the experimental observations very closely. The root mean square errors between the predicted and measured values of rice MC, air temperature, and air RH were less than one percentage point, 5°C, and ten percentage points, respectively. The presented model is intended to provide an accurate mathematical description of cross-flow drying of rough rice, which could facilitate optimization of dryer operation and improvements in dryer design.

The model was also used to predict the distribution of rice kernel material states throughout the drying column. Depending on the initial MC, rice kernels exiting the dryer were predicted to be all in the glassy state, all in the rubbery state, or a mixture in which some kernels were in the glassy state while the remaining kernels were in the rubbery state. Because changes in the material state are believed to produce fissures in rice kernels, such predictions allow the presented model to be used for rice fissuring research.

The accuracy of model predictions could be improved by determining drying constants and EMCs for the particular form of rice being modeled over a comprehensive range of air temperature and RH conditions. Similarly, convective heat transfer coefficients at the rice kernel surface need to be investigated to more accurately describe the heat transfer between grain and air. The presented model does not describe intra-kernel moisture or temperature gradients, which have also been shown to influence fissuring in rice kernels. The inclusion of intra-kernel heat and mass transport phenomena in the dryer model could provide a more detailed understanding of the drying process for developing specific interventions to reduce fissuring, and thereby improve the economic value of rice crops.
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